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A Survey on Different Types of CT Image Reconstruction

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***Abstract*— *Image renovation in CT is a mathematical process that creates images from X-ray projection data gain at many different angles around the patient. Image rebuilding has a basic impact on image worth and therefore on radiation dose. Many techniques have been used to reconstruct the image and the commonly used algorithms are L1 and $L_{1/2.}$ L1 regularization algorithm has been normally used to solve the sparsity constrained problems. To enhance the sparsity constraint for better imaging performance, a promising route is to use the lp norm (0 < p < 1) and solve the lp minimization problem. ½ has been used widely as a replace with for p. In this paper survey the various methods in reconstruction of CT images are discussed.*

***Keywords*—Compressive sampling, half-threshold filtering, discrete gradient transform, pseudo-inverse transform.**

I. INTRODUCTION

A CT scan, also called X-ray computed tomography (X-ray CT) or computerized axial tomography scan (CAT scan), makes use of computer-processed combinations of many X-ray images taken from different angles to produce cross-sectional (tomographic) images (virtual 'slices') of specific areas of a scanned object, allowing the user to see inside the object without cutting. Digital geometry processing is used to generate a three dimensional image of the inside of the object from a large series of two dimensional radiographic images taken around a single axis of rotation. Medical imaging is the most common application of X-ray CT. Its cross-sectional images are used for diagnostic and therapeutic purposes in various medical disciplines. In signal processing, Total variation denoising, also known as total variation regularization is a process, most often used in digital image processing, that has applications in noise removal. It is based on the principle that signals with excessive and possibly spurious detail have high total variation, that is, the integral of the absolute gradient of the signal is high. According to this principle, reducing the total variation of the signal subject to it being a close match to the original signal, removes unwanted detail whilst preserving important details such as edges.

Stimulated by the theory of compressive sampling or compressive sensing (CS) [1], [2], the sparsity based computed tomography (CT) has been a hot topic for various applications such as dose reduction [3]. Because the x-ray decrease coefficient often varies gently within an anatomical component, and large changes are usually confine around borders of anatomical structures, the discrete gradient transform (DGT), a set of finite difference operators, has been widely utilized as a sparsifying action in CS-inspired CT reconstruction such as in, whose L1-norm is also referred to as the total variation (TV) [4], and the equivalent reconstruction techniques are called TV minimization. Recently, the soft-threshold nonlinear filtering [6] was proved to be a converging and efficient algorithm for the L1-norm minimization regularized by a sparsity constraint. Unfortunately, because the DGT is non-invertible, the soft-threshold algorithm cannot be directly applied for TV minimization. To address this challenge, soft-threshold filtering based pseudoinverse transforms for DGT was constructed and applied the soft -threshold filtering technique for image reconstruction from a limited number of Projections [6].

II. WIDELY USED METHODS

A. NON LINEAR VARIATION IN RADIATION DOSE

It is well known that the primary disadvantage of X-ray CT is ionizing radiation which may induce cancers and cause genetic damages with a probability related to the radiation dose. Thus, reducing the dose as low as possible is a general rule for practical medical applications.

A.A. Statistical Assumption in X-ray image reconstruction

The statistical assumption of the signal was the one of the oldest methods used in which, the radiation dose has been reduced. In any SIR method, an optimization criterion is set up based on the likelihood or probability density function (PDF) of the projection data. The maximum-likelihood solution has many desirable properties, one of which is that it is asymptotically the minimum variance solution among all unbiased estimators. The reduced image variance can be traded for reduced radiation dose at the same image noise level, thus achieving dose reduction. An important component of any SIR method therefore is the likelihood function of the raw data. It has been shown that the raw CT data follow a compound Poisson distribution; the exact PDF does not admit an analytic expression hence precludes its use in SIR method development. However, the result of each

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approximation can be varied according to the statistical weightage values and hence might result in varying radiation dose.

A.B. Local ROI reconstruction via FBP and BPF

In the field of medical applications, an efficient way for CT image reconstruction has been to reducing the dose as low as possible, (a general rule for practical medical applications) or reduce the region or volume to be imaged. To reconstruct a long object such as a patient, the usage of Filtered Back Projection (FBP) as well as the formula for Back Filtered Projection (BPF), derived from Katsevich's FBP formula for standard helical cone-beam CT. The key step is to choose a filtering direction based on the general condition. A natural choice is the direction of the generalized PI-segment, also referred as a chord. The modified BPF formula was derived from the FBP by interchanging the order of the Hilbert filtering and back projection operation in Katsevich's FBP formula thus, enabling to reconstruction of the object only from the minimum data. Since the chord discussed here is a 2D locus, we can only obtain the fan beam reconstruction formulas which are not regarded as a best practice for CT image reconstruction since, the Cone beam geometry provides a better angular projection as well as 3D reconstructed image. However introduction of combining both global as well as local datasets for image reconstruction has been considered as an innovative and novel method.

B. NOISE REMOVAL ALGORITHMS

The presence of noise in image is unavoidable. It may be introduced by the image formation process, image recording, image transmission etc. In practice, to estimate a true signal in noise, the most frequently used methods are based on the least square criteria. A constrained minimization algorithm has been derived as a time dependant nonlinear PDE where the constraints are determined by noise statistics. The procedure is sole dependent on the L2 norm form. Experimental results when added Gaussian white noise to the image and after denoising it shows that the procedure beats the human eye. The use of more constraints in this procedure will yield more details of the solution. Instead of L2 norm, L1 norm can also be used. But In comparison to the least square methods, the L1 estimation is non linear and computationally complex. The L1 norm is usually avoided since the variation of some expressions in the algorithm and produces singular distributions as coefficients which can't be handled in a purely algebraic framework.

C. SPARSITY CONSTRAINTS

A sparse approximation is a sparse vector that approximately solves a system of equations. Techniques for finding sparse approximations have found wide use in applications such as image processing, audio processing, biology, and document analysis.

C.A. Reconstruction of Sparse signals via Nonconvex Minimization

Reconstruction of signals with sparse values has always been challenging since we have to reconstruct the image from sparse values (null values). The sparser the equation gets, the easier it is to solve the equation since more linear measurements will be available for a more sparse equation. Several authors have cited that using the traditional sampling theory, (fs>2fm) it is possible to reconstruct exactly a sparse signal from fewer linear measurements. The methods used involve computing the signal of minimum L1 norm among those having the given measurements. To show that by replacing the L1 norm with the Lp norm with p<1, exact reconstruction is possible with substantially fewer measurements [9], [11]. Many researches have been done on the subject of reconstruction of sparse signals from a limited number of linear measurements

From the equation below that defines sparsity

$$X \in C^N, y = \phi_x$$

C is a constant not depending on the K (sparsity of signal x) and N (order of the measurement matrix) such that whenever M > CK logN, the signal x can be reconstructed exactly with very high probability. For better results when the L1 norm is replaced by Lp ($0 < p < 1$), the resulting optimization will not be convex and hence is considered as intractable problem by mathematicians. But recent studies have shown that a local minimizer can be constructed that produce exact reconstruction of sparse signals with many fewer measurements than when $p = 1$.

C.B. Image recovery from Incomplete Fourier measurements

The major problem in imaging applications is the task of trying to reconstruct an image with the smallest possible set of Fourier samples. Compressive sensing points to way of exploiting inherent sparsity in such images for accurate recovery. Traditional CS approaches to this problem consist of

solving total-variation (TV) minimization programs with Fourier measurement constraints or other variations thereof. Since the horizontal and vertical differences of a medical image are each more sparse or compressible than the corresponding TV image, CS methods will be more successful in recovering these differences individually. If the signal or the image is sparse in some domain, then one can reconstruct the signal exactly with significantly fewer Fourier coefficients than originally thought thus, reducing the number of measurements that devices take in order to generate high quality image. By using the fact the Fourier transform of the gradients of an image are precisely equal to a diagonal transformation of the Fourier transform of the original image, we utilize CS methods to directly recover the horizontal and vertical differences of our desired image. Then, we employ one of two techniques for recovering the original image from our edge estimates. This second step can be done by either solving a simple penalized least-square (LS) optimization problem or by utilizing a modified

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Poisson solver, former taking more computation time, latter being considered as an efficient approach. The method solely proposes that instead of reconstructing an image by reducing Total Variation, the exact image can be reconstructed separately reconstructing the gradients and then solving for the images. This allows one to reconstruct the image with a far fewer number of measurements than required by the TV minimization method.

The algorithm used here first modifies the original Fourier measurements to obtain Fourier measurements of the corresponding vertical and horizontal edge images. It then utilizes some algorithm from the suite of CS recovery routines to recover the edge images. Finally, it recovers the original image from the estimates of its edges using one of several specialized integration techniques. Image gradients are estimated from the given fourier observations by the following equations.

$$(\mathcal{F}_\Omega X_x)_k = (1- e^{-2\pi i\omega_{x,k}/N}) (\mathcal{F}_\Omega X)_k$$

$$(\mathcal{F}_\Omega X_y)_k = (1- e^{-2\pi i\omega_{y,k}/N}) (\mathcal{F}_\Omega X)_k$$

The Total Variation technique fails as it needs at least 3 to 5 times the fourier coefficients that of the sparse values. Hence the latter method has been efficient in recovering the images from a lesser number of linear measurements and thus minimizing the computational time for the image recovery.

D. THRESHOLDING ALGORITHMS

Thresholding is a process of converting a grayscale input image to a bi-level image by using an optimal threshold. The purpose of thresholding is to extract those pixels from some image which represent an *object* (either text or other line image data such as graphs, maps). Though the information is binary the pixels represent a range of intensities. Thus the objective of binarization is to mark pixels that belong to true foreground regions with a single intensity and background regions with different intensities. For a thresholding algorithm to be really effective, it should preserve logical and semantic content.

D.A. Fast Iterative Shrinkage Thresholding Algorithm for Linear Inverse Problems

Linear inverse problems arise in a wide range of applications including the image reconstruction. A large body of mathematical algorithms and formulae are required to solve this problem. A classical approach to solve this problem is the least square (LS) approach. But the class of iterative shrinkage thresholding algorithms (ISTA), an extension of classical gradient algorithm, is attractive due to its simplicity. However, because of the usage of the dense matrix data, they tend to converge slowly and hence a worst case complexity result is also to be considered. Recent studies have led to a Fast ISTA keeping the simplicity of the ISTA and a convergence rate far better than the former and an improved complexity result.

D.B. Iterative Hard thresholding & Compressive sensing

Compressive sensing literally means sampling the signal below the Nyquist rate (Sampling theorem) [12], [13]. Most signals in real world are not exactly sparse but have very well approximated values. The iterative hard thresholding is a very simple iterative procedure starting with zero value and uses the iteration

$$X^{n+1} = H_k (X^n + \phi^T (y - \phi_x^n))$$

Where HK(a) is the non-linear operator that sets all but the largest (in magnitude) K elements of a to zero (non-uniqueness issues being avoided using random or deterministic heuristics). It has been proven from studies that the algorithm has near optimal performance whenever the matrix Φ has a small restricted isometry constant δk such that

$$(1 - \delta_k) \|X\|_2^2 \leq \| \phi_x\|^2_2 \leq (1 + \delta_k) \|X\|^2_2$$

holds for all vectors x with no-more than K non-zero elements.

But despite its simplicity, the algorithm is restricted by isometric constants and the matrix needs to be normalized to guarantee the stability. However the use of L2 norm for normalizing the matrix isn't much appreciated in the image processing field since it provides only NP hard problems.

D.C. $L_{1/2}$ Regularization

Recent researches have shown that a promising direction is, instead of using L1 norm forms, the Lp norm ($0 < p < 1$) to improve the sparsity of the image, but leads to Lp regularization problem. Based on a phase diagram study, Xu et al showed that the lesser the value of the p, the more sparser the solution to get. The recent trend is substituting p = ½. The convergence of the L1/2 iterative half thresholding algorithm [9], [10] has been considered as a solution for the L1/2 regularization problem. Hence the formula for half algorithm.

$$X^{(n+1)} = H_{\lambda\mu,1/2}(x^{(n)} - \mu A^T (A\, x^{(n)} - y))$$

The convergence of the half algorithm has been partially analyzed which implies that it can converge to a stationary point when the step size parameter is too small. In the convergence of algorithm, we may be more interested to know if it converges to a global minimizer or a local minimizer point. However the convergence of the algorithm to a local minimizer of L1/2 regularization has been considered more reliable even when if leads to non convex non smooth optimization problem that is difficult to solve faster and efficiently. Through the simulations and comparison with hard thresholding and soft thresholding algorithms, the iterative half thresholding algorithm is adaptive and free from choice of parameters. We have verified the convergence of the proposed algorithm, and applied the algorithm, together with other competitive regularization algorithms, to a series of problems in signal processing. The algorithm is fast, effective, and very efficient for k–sparsity problems. L1/2 regularization shows a significantly stronger sparsity-promoting property than L1

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regularization in the sense that it allows getting more sparse solutions of a problem and recovering a sparse signal from fewer samplings, as compared with L1 regularization.

D.D. Simultaneous Algebraic Reconstruction Method (SART)-Type Half Threshold Filtering

Simultaneous algebraic reconstruction technique (SART) [14] type half-threshold filtering framework to solve the computed tomography reconstruction problem. In the medical imaging field, the discrete gradient transform (DGT) is widely used to define the sparsity. The DGT is noninvertible and it cannot be applied to half-threshold filtering for CT reconstruction. The results show that the SART-type half-threshold filtering algorithms have great potential to improve the reconstructed image quality from few and noisy projections. The main drawback of iterative methods is the relatively high demand for computational time. Several approaches have been developed to accelerate the computation of iterative methods. A popular mechanism is the ordered subsets (OS) [15] algorithms. To improve the convergence speed of OS algorithms and to improve the quality of reconstructed images, use SS-SART adjusts the OS level at each iteration.

III. CONCLUSIONS

A number of techniques for CT image reconstruction have been discussed and compared. There are also many interesting methods that are discussed in this paper and there will be many such methods which can improve the analysis process in one or the other way. Many methods like noise removal algorithm and thresholding algorithm and method are reviewed. While some were simple in format, but taking more computation time, algorithms like fast iterative thresholding algorithms were facing the issue of convergence. The normalization distribution of the equivalent mathematical representation of the raw data has been a key note in the image reconstruction problem. For better results and fast computing, L1/2 thresholding algorithms have been the newest trend. To use simultaneous algebraic reconstruction techniques and sequence subsets simultaneous algebraic reconstruction techniques (SS-SART) for the reconstruction in better improve the quality of images. The reconstruction of CT image from incomplete Fourier data set also has attracted the scientists and has succeeded in the same.

We can see that majority depend on the social networking sites to get their valued information. So by analysing the reviews on these blogs will yield a better understanding of techniques. Each method has faced issues like convergence, sparsity, NP hard problems etc, and solution of each has led to another technique which is considered more reliable than the former.

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